



## Analytic Solution of the Dynamics of Malaria Transmission

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### ABSTRACT

The dynamics of malaria transmission in a population is compartmentalized into the human and vector classes and represented by a set of ordinary differential equations using several known clinical and biological information. The Homotopy Perturbation method which provides solution to various linear and non-linear differential equations was applied to obtain the analytic solution of the model. The variational method was used for simulation to follow the trend of the population in the respective classes over a period of time. The result revealed the possibility of delimitation of the endemic disease through concerted effort.

**Keywords:** Malaria, Ordinary differential equation, Susceptible, Homotopy Perturbation Method,

### INTRODUCTION

Malaria is an ancient infectious disease with a 10,000 years of history (Hempelmann *et al.*, 2009). It is originally from different species of *Plasmodium* ancestors, the disease was first found in three places: West Africa, Central Africa and South East Asia. (Mendis *et al.*, 2001).

In the early period of the last century, the total population of malaria infections has reached a very high level with more than one half of the world's population at significant risk. Since the early 1950s, with the appearance of the effective insecticide DDT and medicines for malaria infections, the disease has almost gone into extinction in North America and most European countries. Presently, deaths caused by malaria mainly happen in Africa. Although, the disease incident rate is still high in India and Southeast Asia, the induced mortality rate is lower in contrast to that in Africa. The reason can be traced to the origin of malaria. The impact of the increasing frequency needs to be carefully evaluated, preferably by mathematical models. It is widely acknowledged that the models can play a crucial role in predicting disease dynamics. The Ross-MacDonald model is the earliest attempt to quantitatively describe the dynamics of malaria transmission at a population level. (Chitnis *et al.*, 2006; Ross, 1992; McDonald, 1957). In this research work, the homotopy perturbation method is used to

find the solution to the dynamics of malaria transmission.

### METHODOLOGY

#### The Homotopy Perturbation Method

The Homotopy Perturbation Method (HPM) was developed by (Ji, 2000). The HPM provides solution to various linear and non-linear differential equations (Jiya, 2010).

The basic idea of the method is by considering the following non-linear differential equation:

$$A(U) - f(r) = 0, \quad r \in \Omega \quad (2.1)$$

with the boundary condition

$$B\left(U, \frac{\partial U}{\partial n}\right) = 0, \quad r \in \Gamma$$

Where A is the general differential operator, B the boundary operator,  $f(r)$  is the analytical function and  $\Gamma$  is the boundary of the domain  $\Omega$ . The A operator can be divided into two major parts L and N been the linear and nonlinear components respectively and (2.1) can be written as:

$$L(U) + N(U) - f(r) = 0, \quad r \in \Omega$$

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The HPM is composed as follows:

$$H(V, h) = (1-h) [L(V) - L(U_0)] + h [A(V) - f(r)] = 0$$

Where  $V(r, P): \Omega \in [0, 1] \rightarrow R$  is an embedded parameter and  $U_0$  is the approximation that satisfies the boundary condition. The solution can then be assumed as power series in  $h$  as follows:

$$V = V_0 + hV_1 + h^2V_2 + \dots$$

With approximation best obtained when:

$$U = \lim_{h \rightarrow 1} = v_0 + hv_1 + h^2v_2 + \dots$$

the rate of convergence majorly depends on the nonlinear operator A (V).

**Solution of the Model Using HPM.**

Consider the system of the equations

$$\frac{dS_h}{dt} - \wedge_h p - (rR_h + (\theta - \phi)v_h - \mu_h - \beta I_v)S_h = 0 \tag{1}$$

$$\frac{dI_h}{dt} - \beta S_h I_v + (\mu_h + \mu_0 + \delta)I_h = 0 \tag{2}$$

$$\frac{dR_h}{dt} - \delta I_h - (\mu_h - rS_h)R_h = 0 \tag{3}$$

$$\frac{dV_h}{dt} - \wedge_h (1-p) - (\phi - \theta)S_h v_h = 0 \tag{4}$$

$$\frac{dS_v}{dt} - \wedge_v + (\mu_v + eI_h)S_v = 0 \tag{5}$$

$$\frac{dI_v}{dt} - eS_v I_h + \mu_v I_v = 0 \tag{6}$$

With initial condition

$$S_h(0) = S_h 0, I_h(0) = I_h 0, R_h(0) = R_h 0, \\ S_v(0) = S_v 0, I_v(0) = I_v 0$$

Where:

$$S_h = \text{Susceptible human}$$

$$I_h = \text{Infected human}$$

$$R_h = \text{Recovered human}$$

$$S_v = \text{Susceptible vector}$$

$$I_v = \text{Infected vector}$$

$$V_h = \text{Therapy class}$$

$$\wedge_v = \text{Birth rate for the vector population}$$

$$\wedge_h = \text{Birth rate for the human population}$$

$$\beta = \text{Transmission rate from } S_h \text{ to } I_v$$

$$\delta = \text{Transmission rate from } I_h \text{ to } R_h$$

$$e = \text{Transmission rate from } S_v \text{ to } I_v$$

$$\theta = \text{Transmission rate from } V_v \text{ to } S_h$$

$$\phi = \text{Transmission rate from } S_h \text{ to } V_h$$

$$\mu_0 = \text{Natural death rate from infection in humans}$$

$$\mu_h = \text{Death resulting from infection in human}$$

$$\mu_v = \text{Natural death resulting from vector population}$$

$$r = \text{Rate of flow from } R_h \text{ to } S_h$$

$$P = \text{Recruitment term.}$$

Applying HPM to (1) we have

$$(1-h) \frac{ds_h}{dt} + h \left[ \frac{ds_h}{dt} - \wedge_h p - (rR_h + (\theta - \phi)v_h - \mu_h - \beta I_v)S_h \right] = 0 \tag{8}$$

Let

$$s_h = c_0 + hc_1 + h^2c_2 + \dots \tag{9}$$

$$I_h = e_0 + he_1 + h^2e_2 + \dots \tag{10}$$

$$R_h = w_0 + hw_1 + h^2w_2 + \dots \tag{11}$$

$$V_h = k_0 + hk_1 + h^2k_2 + \dots \tag{12}$$

$$S_v = x_0 + hx_1 + h^2x_2 + \dots \tag{13}$$

$$I_v = y_0 + hy_1 + h^2y_2 + \dots \tag{14}$$

Put equations (10) to (14) in (8)

$$(1-h)(c'_0 + hc'_1 + h^2c'_2 + \dots) + h \left[ \begin{array}{l} (c'_0 + hc'_1 + h^2c'_2 + \dots) - \wedge_h p - (r(w_0 + hw_1 + h^2w_2 + \dots) + \\ (\theta - \phi)(k_0 + hk_1 + h^2k_2 + \dots) - \mu_h - \beta(y_0 + hy_1 + h^2y_2 + \dots)) \\ c_0 + hc_1 + h^2c_2 + \dots \end{array} \right] = 0 \tag{15}$$

Simplifying (15) and collecting the coefficient of powers of h, we have

$$h^0 : c'_0 = 0 \tag{16}$$

$$h^1 : c'_1 - \wedge_h prw_0 + (\theta - \phi)k_0 - \mu_h - \beta y_0 c_0 \tag{17}$$

$$h^2 : c'_2 - \wedge_h pw_1 + (\theta - \phi)k_1 - \beta y_0 c_1 - \beta y_1 c_0 \tag{18}$$

$$h^3 : -\wedge_h pw_2 + (\theta - \phi)k_2 - \beta y_0 c_2 - \beta y_1 c_1 - \beta y_2 c_0 \tag{19}$$

Substitute equations (9), (10) and (14) in to (22)

$$(1-h)(e'_0 + he'_1 + h^2e'_2 + \dots) + h \left[ \begin{array}{l} (e'_0 + he'_1 + h^2e'_2 + \dots) - \beta \\ (c_0 + hc_1 + h^2c_2 + \dots)(y_0 + hy_1 + h^2y_2 + \dots) + (\mu_h + \mu_0 + \delta) \\ (e_0 + he_1 + h^2e_2 + \dots) \end{array} \right] = 0 \tag{23}$$

Simplifying equation (23) and collecting the coefficients of powers of h we have

$$h^0 : e'_0 = 0 \tag{24}$$

$$h^1 : e'_1 - \beta c_0 y_0 + (\mu_h + \mu_0 + \delta)e_0 \tag{25}$$

$$h^2 : e'_2 - \beta c_0 y_1 - \beta c_1 y_0 + (\mu_h + \mu_0 + \delta)e_1 \tag{26}$$

$$h^3 : -\beta c_0 y_2 - \beta c_1 y_1 - \beta c_2 y_0 + (\mu_h + \mu_0 + \delta)e_2 \tag{27}$$

Substitute equations (9), (10) and (11) in (30)

$$(1-h)(w'_0 + hw'_1 + h^2w'_2 + \dots) + h \left[ \begin{array}{l} (w'_0 + hw'_1 + h^2w'_2 + \dots) - (\delta(e_0 + he_1 + h^2e_2 + \dots) - \mu_h - \\ r(c_0 + hc_1 + h^2c_2 + \dots))(w_0 + hw_1 + h^2w_2 + \dots) \end{array} \right] = 0 \tag{31}$$

Simplifying equation (31) and collecting the coefficients of powers of h, we have,

$$h^0 : w'_0 = 0 \tag{32}$$

$$h^1 : w'_1 - \delta e_0 w_0 - \mu_h w_0 - rc_0 w_0 \tag{33}$$

$$h^2 : w'_2 - \delta e_0 w_1 - \delta e_1 w_0 - \mu_h w_1 - rc_0 w_1 \tag{34}$$

$$h^3 : -\delta e_0 w_2 - \delta e_1 w_1 - \delta e_2 w_0 - \mu_h w_2 - rc_0 w_2 - rc_1 w_1 - rc_2 w_0 \tag{35}$$

$$h^4 : -\beta y_1 c_2 - \beta y_2 c_1 \tag{20}$$

$$h^5 : -\beta y_2 c_2 \tag{21}$$

Applying HPM to (2) we have

$$(1-h) \frac{dI_h}{dt} + h \left[ \frac{dI_h}{dt} - \beta s_h I_v + (\mu_h + \mu_0 + \delta) I_h \right] = 0 \tag{22}$$

$$h^4 : -\beta c_0 y_2 - \beta c_2 y_1 \tag{28}$$

$$h^5 : -\beta c_2 y_2 \tag{29}$$

Applying HPM to equation (3) we have

$$(1-h) \frac{dR_h}{dt} + h \left[ \frac{dR_h}{dt} - (\delta I_h - \mu_h - rs_h) R_h \right] = 0 \tag{30}$$

$$h^4 : -\delta e_1 w_2 - \delta e_2 w_1 - rc_1 w_2 - rc_2 w_1 \tag{36}$$

$$h^5 : -rc_2 w_2 \tag{37}$$

Applying HPM to equation (4) we have

$$(1-h) \frac{dv_h}{dt} + h \left[ \frac{dv_h}{dt} - \wedge_h (1-p) - (\theta - \phi) s_h v_h \right] = 0 \tag{38}$$

Substituting equations (9) and (12) in to (38)

$$(1-h)(k'_0 + hk'_1 + h^2k'_2 + \dots) + h \left[ \begin{matrix} (k'_0 + hk'_1 + h^2k'_2 + \dots) - \wedge_h(1-p) - \\ (\theta - \phi)(c_0 + hc_1 + h^2c_2 + \dots)(k_0 + hk_1 + h^2k_2 + \dots) \end{matrix} \right] = 0 \tag{39}$$

Simplifying equation (39) and collecting the coefficients of powers of h, we have

$$h^0 : k'_0 = 0 \tag{40}$$

$$h^1 : k'_1 - \wedge_h(1-p) - (\theta - \phi)c_0k_0 \tag{41}$$

$$h^2 : k'_2 - (\theta - \phi)c_0k_1 - (\theta - \phi)c_1k_0 \tag{42}$$

$$h^3 : -(\theta - \phi)c_0k_2 - (\theta - \phi)c_1k_1 - (\theta - \phi)c_2k_0 \tag{43}$$

$$h^4 : -(\theta - \phi)c_1k_2 - (\theta - \phi)c_2k_1 \tag{44}$$

$$h^5 : -(\theta - \phi)c_2k_2 \tag{45}$$

Applying HPM to equation (5)

$$(1-h) \frac{ds_v}{dt} + h \left[ \frac{ds_v}{dt} - \wedge_v(\mu_v + eI_h)s_v \right] = 0 \tag{46}$$

$$(46)$$

Substituting equation (10) and (13) in to (46)

$$(1-h)(x'_0 + hx'_1 + h^2x'_2 + \dots) + h \left[ \begin{matrix} (x'_1 + hx'_2 + h^2x'_3 + \dots) - \wedge_v(\mu_v + \\ e(e_0 + he_1 + h^2e_2 + \dots))(x_0 + hx_1 + h^2x_2 + \dots) \end{matrix} \right] = 0 \tag{47}$$

Simplifying equation (47) and collecting the coefficients of powers of h we have

$$h^0 : x'_0 = 0 \tag{48}$$

$$h^1 : x'_1 - \wedge_v\mu_vx_0 - ec_0x_0 = 0 \tag{49}$$

$$h^2 : x'_2 - \wedge_v\mu_vx_1 - ec_0x_1 - ex_0e_1 = 0 \tag{50}$$

$$h^3 : -\wedge_v\mu_vx_2 - ec_0x_2 - ex_1e_1 - ex_0e_2 = 0 \tag{51}$$

$$h^4 : -ex_2e_1 - ex_1e_2 = 0 \tag{52}$$

$$h^5 : -ex_2e_2 = 0 \tag{53}$$

Applying HPM to equation (6)

$$(1-h) \frac{dI_v}{dt} + h \left[ \frac{dI_v}{dt} - es_vI_h + \mu_vI_v \right] = 0 \tag{54}$$

Substitute equations (10), (13) and (14) in to (54)

$$(1-h)(y'_0 + hy'_1 + h^2y'_2 + \dots) + h \left[ \begin{matrix} (y'_1 + hy'_2 + h^2y'_3 + \dots) - \\ e(x_0 + hx_1 + h^2x_2 + \dots)(e_0 + he_1 + h^2e_2 + \dots) + \\ \mu_v(y_0 + hy_1 + h^2y_2 + \dots) \end{matrix} \right] = 0 \tag{55}$$

Simplifying equation (55) and collecting the coefficients of the powers of h we have

$$h^0 : y'_0 = 0 \tag{56}$$

$$h^1 : y'_1 + ex_0e_0 + \mu_vy_0 = 0 \tag{57}$$

$$h^2 : y'_2 + x_0e_1 + x_1e_0 + \mu_vy_1 = 0 \tag{58}$$

$$h^3 : e_2x_0 + e_1x_1 + e_0x_2 = 0 \tag{59}$$

$$h^4 : x_1e_2 = 0 \tag{60}$$

$$h^5 : x_2e_1 + x_2e_2 = 0 \tag{61}$$

From equation (24)

$$e'_0 = 0 \tag{62}$$

Integrating we have

$$\int e'_0 = \int 0 \tag{63}$$

$$e_0 = A \tag{64}$$

Applying initial condition

$$e_0(0) = I_h^0 = e_0 \tag{65}$$

$$A = I_h^0 \tag{66}$$

$$\text{Thus, } e_0 = I_h^0 \tag{67}$$

From equation (16)

$$c'_0 = 0 \tag{68}$$

Integrating we have

$$\int c'_0 = \int 0 \quad (69)$$

$$c_0 = B \quad (70)$$

Applying initial condition

$$c_0(0) = s_h^0 = c_0 \quad (71)$$

$$B = s_h^0 \quad (72)$$

$$\text{Thus } c_0 = s_h^0 \quad (73)$$

From equation (32)

$$w'_0 = 0 \quad (74)$$

$$\text{Integrating we have} \quad (75)$$

$$\int w'_0 = \int 0 \quad (76)$$

$$w_0 = C \quad (77)$$

Applying initial condition

$$w_0(0) = R_h^0 = w_0 \quad (78)$$

$$C = R_h^0 \quad (79)$$

$$\text{Thus } w_0 = R_h^0 \quad (80)$$

From equation (40)

$$k'_0 = 0 \quad (81)$$

Integrating we have

$$\int k'_0 = \int 0 \quad (81)$$

$$k_0 = D \quad (82)$$

Applying initial condition

$$k_0(0) = v_h^0 = k_0 \quad (83)$$

$$D = v_h^0 \quad (84)$$

$$\text{Thus, } k_0 = v_h^0 \quad (85)$$

From equation (48)

$$x'_0 = 0 \quad (86)$$

Integrating we have

$$\int x'_0 = \int 0 \quad (87)$$

$$x_0 = E \quad (88)$$

Applying initial condition

$$x_0(0) = s_v^0 = x_0 \quad (89)$$

$$E = s_v^0 \quad (90)$$

$$\text{Thus, } x_0 = s_v^0 \quad (91)$$

From equation (56)

$$y'_0 = 0 \quad (92)$$

Integrating we have

$$\int y'_0 = \int 0 \quad (93)$$

$$y_0 = F \quad (94)$$

Applying initial condition

$$y_0(0) = I_v^0 = y_0 \quad (95)$$

$$F = I_v^0 \quad (96)$$

$$\text{Thus } y_0 = I_v^0 \quad (97)$$

Similarly From equation (17)

$$c'_1 = \wedge_h prw_0 - (\theta - \phi)k_0 + \mu_h + \beta y_0 c_0 \quad (98)$$

Integrating we have

$$\int c'_1 = \int (\wedge_h prw_0 - (\theta - \phi)k_0 + \mu_h + \beta y_0 c_0) \quad (99)$$

$$\int c'_1 = \int (\wedge_h prw_0 - (\theta - \phi)k_0 + \mu_h + \beta y_0 c_0) dt \quad (100)$$

$$c_1 = (\wedge_h prw_0 - (\theta - \phi)k_0 + \mu_h + \beta y_0 c_0)t + G \quad (101)$$

where G is constant.

Applying the initial condition

$$c_1(0) = 0$$

$$c_1 = (\wedge_h prw_0 - (\theta - \phi)k_0 + \mu_h + \beta y_0 c_0)t \quad (102)$$

Consequently

$$c_1 = (\wedge_h pr(R_h^0) - (\theta - \phi)v_h^0 + \mu_h + \beta I_v^0 s_h^0)t \quad (103)$$

From equation (25)

$$e'_1 = \beta c_0 y_0 - \mu_h + \mu_0 + \delta \quad (104)$$

Integrating we have

$$\int de_1 = \int (\beta c_0 y_0 - \mu_h + \mu_0 + \delta) dt \quad (105)$$

$$e_1 = (\beta c_0 y_0 - (\mu_h + \mu_0 + \delta)e_0)t + H \quad (106)$$

Where H is constant

Applying the initial condition

$$e_1(0) = 0 \quad (107)$$

$$e_1 = (\beta c_0 y_0 - (\mu_h + \mu_0 + \delta)e_0)t \quad (108)$$

Consequently

$$e_1 = (\beta(s_h^0)(I_h^0) - (\mu_h + \mu_0 + \delta)e_0)t \quad (109)$$

From equation (33)

$$w'_1 = \delta e_0 w_0 + \mu_h w_0 + r c_0 w_0 \quad (110)$$

Integrating gives

$$w_1 = (\delta e_0 w_0 + \mu_h w_0 + r c_0 w_0)t + I \quad (111)$$

Applying initial condition

$$w_1 = (\delta e_0 w_0 + \mu_h w_0 + r c_0 w_0)t \quad (112)$$

Hence

$$w_1 = (\delta I_h^0 R_h^0 + \mu_h R_h^0 + r s_h^0 R_h^0) t \tag{113}$$

Integrating equation (41) gives

$$k_1' = \wedge_h (1 - p) + (\theta - \phi) c_0 k_0$$

$$k_1 = (\wedge_h (1 - p) + (\theta - \phi) c_0 k_0) t + J \tag{114}$$

Applying initial condition

$$k_1(0) = 0 \tag{115}$$

$$k_1 = (\wedge_h (1 - p) + (\theta - \phi) c_0 k_0) t \tag{116}$$

Consequently

$$k_1 = (\wedge_h (1 - p) + (\theta - \phi) s_h^0 v_h^0) t \tag{117}$$

From equation (49)

$$x_1' = \wedge_v \mu_v X_0 + e c_0 X_0 \tag{118}$$

Integrating gives

$$x_1 = (\wedge_v \mu_v x_0 + e c_0 x_0) t + K \tag{119}$$

Applying initial condition

$$x_1(0) = 0 \tag{120}$$

$$c_2' = \left[ \begin{aligned} &\wedge_h p (\delta I_h^0 R_h^0 + \mu_h R_h^0 + r s_h^0 R_h^0) - (\theta - \phi) (\wedge_h (1 - p) + (\theta - \phi) s_h^0 v_h^0) + \beta I_v^0 (\wedge_h p r R_h^0) \\ &- (\theta - \phi) v_h^0 + \mu_h + \beta (I_v^0 s_h^0 + \beta (-e s_v^0 I_h^0 - \mu_v I_v^0 s_h^0)) \end{aligned} \right] t \tag{129}$$

Integrating and applying the initial condition to (129)

$$c_2 = \left[ \begin{aligned} &\wedge_h p (\delta I_h^0 (R_h^0 + \mu_h R_h^0 + r s_h^0 R_h^0) - (\theta - \phi) (\wedge_h (1 - p) + (\theta - \phi) s_h^0 v_h^0) + \beta I_v^0 (\wedge_h p r R_h^0) \\ &- (\theta - \phi) v_h^0 + \mu_h + \beta I_v^0 s_h^0) + \beta (-e s_v^0 I_h^0 - \mu_v I_v^0 s_h^0) \end{aligned} \right] \frac{t^2}{2} \tag{130}$$

From equation (9)

$$s_h(t) = c_0 + h c_1 + h^2 c_2 \tag{131}$$

Taking the Lim  $h \rightarrow 1$ , gives

$$s_h(t) = c_0 + c_1 + c_2 \tag{132}$$

Substitute equations of  $c_0, c_1$  and  $c_2$  in to (132)

$$x_1 = (\wedge_v \mu_v x_0 + e c_0 x_0) t \tag{121}$$

Consequently

$$x_1 = (\wedge_v \mu_v (s_v^0) + e s_h^0 s_v^0) t \tag{122}$$

From equation (57)

$$y_1' = -e x_0 e_0 - \mu_v y_0 = 0 \tag{123}$$

And

$$y_1 = (-e x_0 e_0 - \mu_v y_0) t + L \tag{124}$$

Applying initial condition

$$y_1(0) = 0 \tag{125}$$

$$y_1 = (-e x_0 e_0 - \mu_v y_0) t \tag{126}$$

And so

$$y_1 = (-e s_v^0 I_h^0 - \mu_v I_v^0) t \tag{127}$$

Similarly,

From equation (18)

$$c_2' = \wedge_h p w_1 - (\theta - \phi) k_1 + \beta y_0 c_1 + \beta y_1 c_0 \tag{128}$$

Therefore

$$s_h(t) = s_h^0 + (\wedge_h p r R_h^0 - (\theta - \phi) v_h^0 + \mu_h + \beta I_v^0 s_h^0) t + \left[ \begin{aligned} &\wedge_h p (\delta I_h^0 R_h^0 + \mu_h R_h^0 + r s_h^0 R_h^0) \\ &- (\theta - \phi) (\wedge_h (1 - p) + (\theta - \phi) s_h^0 v_h^0) + \\ &\beta I_v^0 (\wedge_h p r R_h^0) \\ &- (\theta - \phi) v_h^0 + \mu_h + \beta I_v^0 s_h^0) + \\ &\beta (-e s_v^0 I_h^0 - \mu_v I_v^0 s_h^0) \end{aligned} \right] \frac{t^2}{2} \tag{133}$$

From equation (26)

$$e_2' = \beta c_0 y_1 + \beta c_1 y_0 - (\mu_h + \mu_0 + \delta) e_1 \tag{134}$$

Hence

$$e_2' = \left[ \begin{aligned} &\beta (s_h^0) - e (s_v^0) (I_v^0) - \mu_v (I_v^0) + \beta (\wedge_h p r (R_h^0) - (\theta - \phi) v_h^0 + \mu_h + \beta (s_v^0) (I_v^0) - \\ &(\mu_h + \mu_0 + \delta) (\beta (s_h^0) (I_v^0) - \mu_h + \mu_0 + \delta) \end{aligned} \right] t \tag{135}$$

Integrating and applying the initial condition

$$e_2(0) = 0 \text{ to (135)}$$

$$e_2 = \left[ \begin{array}{l} \beta(s_h^0) - e(s_v^0)(I_v^0) - \mu_v(I_v^0) + \beta(\wedge_h pr(R_h^0) - (\theta - \phi)v_h^0 + \mu_h + \beta(s_v^0)(I_v^0) - \\ (\mu_h + \mu_0 + \delta)(\beta(s_h^0)(I_v^0) - (\mu_h + \mu_0 + \delta)e_0 \end{array} \right] \frac{t^2}{2} \quad (136)$$

Now from equation (10)

$$I_h(t) = e_0 + he_1 + h^2e_2... \quad (137)$$

Taking the Lim  $h \rightarrow 1$ , we have

$$I_h(t) = e_0 + e_1 + e_2$$

Substituting  $e_0, e_1$  and  $e_2$  in to (137)

$$I_h(t) = (I_h^0) + (\beta(s_h^0)(I_h^0) - (\mu_h + \mu_0 + \delta)I_h^0)t + \left[ \begin{array}{l} \beta(s_h^0) - e(s_v^0)(I_v^0) - \mu_v(I_v^0) + \beta(\wedge_h pr(R_h^0) - (\theta - \phi)v_h^0 + \mu_h + \beta(s_v^0)(I_v^0) - \\ (\mu_h + \mu_0 + \delta)(\beta(s_h^0)(I_v^0) - (\mu_h + \mu_0 + \delta)I_h^0 \end{array} \right] \frac{t^2}{2} \quad (138)$$

From equation (34)

$$w_2' = \delta e_0 w_1 + \delta e_1 w_0 + \mu_h w_1 + rc_0 w_1 \quad (139)$$

Hence

$$w_2' = \left[ \begin{array}{l} \delta(I_h^0)(\delta(I_h^0)(R_h^0) + \mu_h(R_h^0) + r(s_h^0)(R_h^0) + \delta(\beta(s_h^0)(I_v^0) - \mu_h + \mu_0 + \delta)(R_h^0) + \\ \mu_h(\delta(I_h^0)(R_h^0) + \mu_h(R_h^0) + r(s_h^0)(R_h^0)) + r(s_h^0)(\delta(I_h^0)(R_h^0) + \mu_h(R_h^0) + r(s_h^0)(R_h^0)) \end{array} \right] t \quad (140)$$

Integrating and applying the initial condition  $w_2(0) = 0$  to (140)

$$w_2 = \left[ \begin{array}{l} \delta(I_h^0)(\delta(I_h^0)(R_h^0) + \mu_h(R_h^0) + r(s_h^0)(R_h^0) + \delta(\beta(s_h^0)(I_v^0) - \mu_h + \mu_0 + \delta)(R_h^0) + \\ \mu_h(\delta(I_h^0)(R_h^0) + \mu_h(R_h^0) + r(s_h^0)(R_h^0)) + r(s_h^0)(\delta(I_h^0)(R_h^0) + \mu_h(R_h^0) + r(s_h^0)(R_h^0)) \end{array} \right] \frac{t^2}{2} \quad (141)$$

Now from equation (11)

$$R_h(t) = w_0 + hw_1 + h^2w_2... \quad (142)$$

Taking the Lim  $h \rightarrow 1$ ,

$$R_h(t) = w_0 + w_1 + w_2 \quad (143)$$

Substituting for  $w_0, w_1$  and  $w_2$  in to (143)

$$R_h(t) = (R_h^0) + (\delta(I_h^0)(R_h^0) + \mu_h(R_h^0) + r(s_h^0)(R_h^0))t + \left[ \begin{array}{l} \delta(I_h^0)(\delta(I_h^0)(R_h^0) + \mu_h(R_h^0) + r(s_h^0)(R_h^0) + \delta(\beta(s_h^0)(I_v^0)) \\ - \mu_h + \mu_0 + \delta)(R_h^0) \\ \mu_h(\delta(I_h^0)(R_h^0) + \mu_h(R_h^0) + r(s_h^0)(R_h^0)) + \\ r(s_h^0)(\delta(I_h^0)(R_h^0) + \mu_h(R_h^0) + r(s_h^0)(R_h^0)) \end{array} \right] \frac{t^2}{2} \quad (144)$$

From equation (42)

$$k_2' = (\theta - \phi)c_0k_1 + (\theta - \phi)c_1k_0 \quad (145)$$

Hence

$$k_2' = \left[ \begin{array}{l} (\theta - \phi)(s_h^0)(\wedge_h(1-p) - (\theta - \phi)(s_h^0)(v_h^0)) + (\theta - \phi)(\wedge_h pr(R_h^0) - (\theta - \phi)v_h^0 + \mu_h + \\ \beta((I_v^0)(s_h^0))(v_h^0) \end{array} \right] t \quad (146)$$

Integrating and applying initial condition  $k_2(0) = 0$  to (146)

$$k_2 = \left[ \begin{array}{l} (\theta - \phi)(s_h^0)(\wedge_h(1-p) - (\theta - \phi)(s_h^0)(v_h^0)) + (\theta - \phi)(\wedge_h pr(R_h^0) - (\theta - \phi)v_h^0 + \mu_h + \\ \beta((I_v^0)(s_h^0))(v_h^0) \end{array} \right] \frac{t^2}{2} \quad (147)$$

Now from equation (12)

$$v_h(t) = k_0 + hk_1 + h^2k_2 \quad (148)$$

Taking the Lim  $h \rightarrow 1$ , gives

$$v_h(t) = k_0 + k_1 + k_2 \quad (149)$$

Substituting for  $k_0$ ,  $k_1$  and  $k_2$  in to (149)

$$v_h(t) = v_h^0 + (\wedge_h(1-p) + (\theta - \phi)(s_h^0)(v_h^0))t + \left[ \begin{array}{l} (\theta - \phi)(s_h^0)(\wedge_h(1-p) - (\theta - \phi)(s_h^0)(v_h^0)) + (\theta - \phi)(\wedge_h pr(R_h^0) - (\theta - \phi)v_h^0 + \mu_h + \\ \beta((I_v^0)(s_h^0))(v_h^0) \end{array} \right] \frac{t^2}{2} \quad (150)$$

From equation (50)

$$x_2' = \wedge_v \mu_v x_1 + e c_0 x_1 + e x_0 e_1 \quad (151)$$

Consequently

$$x_2' = \left[ \begin{array}{l} \wedge_v \mu_v (\wedge_v \mu_v (s_v^0) + e(s_h^0)(s_v^0)) + e(s_h^0)(\wedge_v \mu_v (\wedge_v \mu_v (s_v^0) + e(s_h^0)(s_v^0)) \\ + e(s_v^0)(\beta(s_h^0)(I_h^0) - \mu_h + \mu_0 + \delta) \end{array} \right] t \quad (152)$$

Integrating (151) and applying initial condition  $x_2(0) = 0$



$$x_2 = \left[ \begin{array}{l} \wedge_v \mu_v (\wedge_v \mu_v (s_v^0) + e(s_h^0)(s_v^0)) + e(s_h^0)(\wedge_v \mu_v (\wedge_v \mu_v (s_v^0) + e(s_h^0)(s_v^0))) \\ + e(s_v^0)(\beta(s_h^0)(I_h^0) - \mu_h + \mu_0 + \delta) \end{array} \right] \frac{t^2}{2} \quad (152)$$

Now from equation (13)

$$S_v(t) = x_0 + hx_1 + h^2x_2 + \dots \quad (153)$$

Taking the Lim  $h \rightarrow 1$ ,

$$S_v(t) = x_0 + x_1 + x_2 + \dots \quad (154)$$

Substitute equations of  $x_0, x_1, x_2$  in to (154)

$$S_v(t) = (s_v^0) + (\wedge_v \mu_v (s_v^0) + e(s_h^0)(s_v^0))t + \left[ \begin{array}{l} \wedge_v \mu_v (\wedge_v \mu_v (s_v^0) + e(s_h^0)(s_v^0)) + e(s_h^0)(\wedge_v \mu_v (\wedge_v \mu_v (s_v^0) + e(s_h^0)(s_v^0))) \\ + e(s_v^0)(\beta(s_h^0)(I_h^0) - \mu_h + \mu_0 + \delta) \end{array} \right] \frac{t^2}{2} \quad (155)$$

From equation (58)

$$y_2' = -x_0e_1 - x_1e_0 - \mu_v y_1 = 0 \quad (156)$$

Hence

$$y_2' = \left[ \begin{array}{l} -(s_v^0)(\beta(s_h^0)(I_h^0) - \mu_h + \mu_0 + \delta) - (\wedge_v \mu_v (s_v^0) + e(s_h^0)(s_v^0)(I_h^0) - \\ \mu_v (-e(s_v^0)(I_h^0) - \mu_v(I_v^0)) \end{array} \right] t \quad (157)$$

Integrating (157) and applying initial condition  $y_2(0) = 0$

$$y_2 = \left[ \begin{array}{l} -(s_v^0)(\beta(s_h^0)(I_h^0) - \mu_h + \mu_0 + \delta) - (\wedge_v \mu_v (s_v^0) + e(s_h^0)(s_v^0)(I_h^0) - \\ \mu_v (-e(s_v^0)(I_h^0) - \mu_v(I_v^0)) \end{array} \right] \frac{t^2}{2} \quad (158)$$

Now from equation (158)

$$I_v(t) = y_0 + hy_1 + h^2y_2 + \dots \quad (159)$$

Taking the Lim  $h \rightarrow 1$ ,

$$I_v(t) = y_0 + y_1 + y_2 + \dots \quad (160)$$

Substituting for  $y_0, y_1, y_2$  in (160)

$$I_v(t) = I_v^0 + (-e(s_v^0)(I_h^0) - \mu_v(I_v^0))t + \left[ \begin{array}{l} -(s_v^0)(\beta(s_h^0)(I_h^0) - \mu_h + \mu_0 + \delta) - (\wedge_v \mu_v (s_v^0) + e(s_h^0)(s_v^0)(I_h^0) - \\ \mu_v (-e(s_v^0)(I_h^0) - \mu_v(I_v^0)) \end{array} \right] \frac{t^2}{2} \quad (161)$$

Therefore, the general analytical solution to the model is

$$s_h(t) = s_h^0 + (\wedge_h prR_h^0 - (\theta - \phi)v_h^0 + \mu_h + \beta I_v^0 s_h^0)t + \left[ \begin{array}{l} \wedge_h p(\delta I_h^0 R_h^0 + \mu_h R_h^0 + rs_h^0 R_h^0) \\ - (\theta - \phi)(\wedge_h(1 - p) + (\theta - \phi)s_h^0 v_h^0 + \\ \beta I_h^0 (\wedge_h prR_h^0) \\ - (\theta - \phi)v_h^0 + \mu_h + \beta I_h^0 s_h^0) + \\ \beta(-es_h^0 I_h^0 - \mu_v I_h^0 s_h^0) \end{array} \right] \frac{t^2}{2} \quad (162)$$

$$I_h(t) = (I_h^0) + (\beta(s_h^0)(I_h^0) - (\mu_h + \mu_0 + \delta)I_h^0)t + \left[ \beta(s_h^0) - e(s_h^0)(I_h^0) - \mu_v(I_h^0) + \beta(\wedge_h pr(R_h^0) - (\theta - \phi)v_h^0 + \mu_h + \beta(s_h^0)(I_h^0) - (\mu_h + \mu_0 + \delta)(\beta(s_h^0)(I_h^0) - (\mu_h + \mu_0 + \delta)I_h^0) \right] \frac{t^2}{2} \quad (163)$$

$$R_h(t) = (R_h^0) + (\delta(I_h^0)(R_h^0) + \mu_h(R_h^0) + r(s_h^0)(R_h^0))t + \left[ \delta(I_h^0)(\delta(I_h^0)(R_h^0) + \mu_h(R_h^0) + r(s_h^0)(R_h^0) + \delta(\beta(s_h^0)(I_h^0) - \mu_h + \mu_0 + \delta)(R_h^0) + \mu_h(\delta(I_h^0)(R_h^0) + \mu_h(R_h^0) + r(s_h^0)(R_h^0)) + r(s_h^0)(\delta(I_h^0)(R_h^0) + \mu_h(R_h^0) + r(s_h^0)(R_h^0)) \right] \frac{t^2}{2} \quad (164)$$

$$v_h(t) = v_h^0 + (\wedge_h(1-p) + (\theta - \phi)(s_h^0)(v_h^0))t + \left[ (\theta - \phi)(s_h^0)(\wedge_h(1-p) - (\theta - \phi)(s_h^0)(v_h^0)) + (\theta - \phi)(\wedge_h pr(R_h^0) - (\theta - \phi)v_h^0 + \mu_h) + \beta((I_v^0)(s_h^0))(v_h^0) \right] \frac{t^2}{2} \quad (165)$$

$$S_v(t) = (s_h^0) + (\wedge_v \mu_v(s_v^0) + e(s_h^0)(s_v^0))t + \left[ \wedge_v \mu_v(\wedge_v \mu_v(s_v^0) + e(s_h^0)(s_v^0)) + e(s_h^0)(\wedge_v \mu_v(\wedge_v \mu_v(s_v^0) + e(s_h^0)(s_v^0)) + e(s_h^0)(\beta(s_h^0)(I_h^0) - (\mu_h + \mu_0 + \delta)I_h^0) \right] \frac{t^2}{2} \quad (166)$$

$$I_v(t) = I_v^0 + (-e(s_v^0)(I_h^0) - \mu_v(I_h^0))t + \left[ -(s_v^0)(\beta(s_h^0)(I_h^0) - (\mu_h + \mu_0 + \delta)I_h^0 - (\wedge_v \mu_v(s_v^0) + e(s_h^0)(s_v^0)(I_h^0) - \mu_v(-e(s_v^0)(I_h^0) - \mu_v(I_v^0)) \right] \frac{t^2}{2} \quad (167)$$

**Simulation Using Results from HPM**

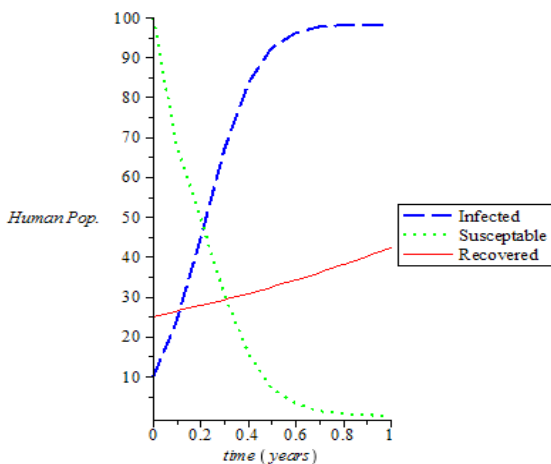


Fig. 1.1 Human Population Over Time in Years for

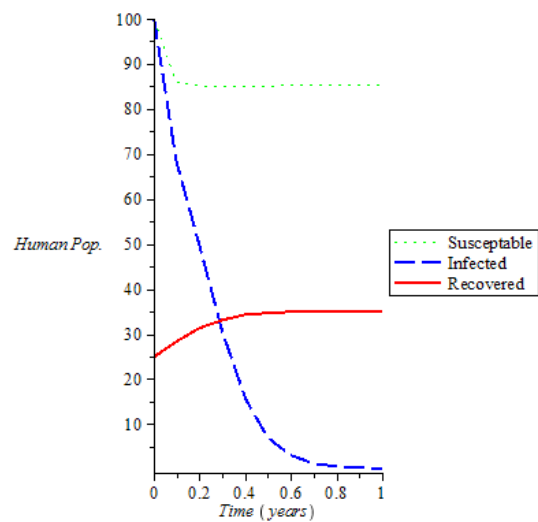


Fig. 1.2 Human Population Over Time in Years

Using the variation method, the simulation is carried out using the data as in Olaniyi and Obabiyi (2013). Other parameters were assumed for lack of availability of data.

$\Lambda_h = 0.04, \beta = 0.001, e = 0.07, \delta = 0.5, \theta = 0.3$   
 $\phi = 0.32, r = 0.00137, \mu_h = 0.548, \mu_0 = 0.02,$   
 $\mu_v = 0.0429, p = 0.002, \Lambda_v = 0.0515,$

$\Lambda_h = 0.04, \beta = 0.1, e = 0.07, \delta = 0.01, \theta = 0.08$   
 $\phi = 0.32, r = 0.00137, \mu_h = 0.548, \mu_0 = 0.02,$   
 $\mu_v = 0.0429, p = 0.002, \Lambda_v = 0.0515,$

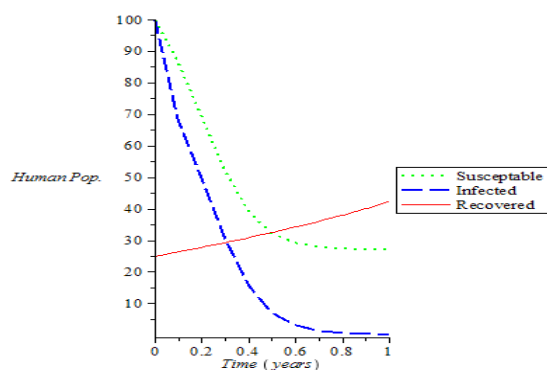


Fig. 1.3 Human Population Over Time in Years

$$\Lambda_h = 0.04, \beta = 0.01, e = 0.07, \delta = 0.2, \theta = 0.2$$

$$\phi = 0.32, r = 0.00137, \mu_h = 0.548, \mu_0 = 0.02,$$

$$\mu_v = 0.0429, p = 0.002, \Lambda_v = 0.0515,$$

## DISCUSSION

Fig. 1.1 depicts sharp decrease in the susceptible population and increase in the infected class. These two segments of the population reached deteriorating levels at time  $t=0$ . The initial population of susceptible almost shrinks to zero after the first year, only to be overtaken by the infected population. This is so because the treatment rate is very low as only slight increase in the recovered class is noticeable. Fig. 1.2 is a healthier population as far as the endemic is concern. The infected population diminishes faster over the period of time. Thought at the onset, the susceptible population is on the decrease but maintains steady state after time  $t=0.2$ . The recovered class is on the increase as a result of inflow from the infected class. In fig. 1.3, the population is significantly infected before attempt to fight the scourge. After time  $t=0.4$ , serious improvement is noticed with the susceptible and infected classes heading towards increase while the infected is almost wiped out at time  $t=1$ .

## Conclusion

The Homotopy Perturbation Method was applied to obtain solution to the set of ordinary differential equations derived from the dynamics of malaria transmission. The variational method was also applied to obtain

the graphs from which future trend of the endemic is revealed. In this case, the transmission can be controlled. The research work further affirms the usefulness of the HPM in finding solution to a set of differential equations.

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